Possibilistic Rule Evaluation
A case study in facial expression analysis

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Abstract. Fuzzy rule systems are an important element in the arsenal of automated control, as they are able to process the input provided by sensors and provide their output in very small times. Lately, there has been augmented interest in utilizing fuzzy rule systems in a wider range of applications, due to the intuitive way in which they represent and utilize knowledge. In these contexts, input is not always readily available from a sensor but often provided as the questionable output of another expert system or even totally missing. In this paper we propose a novel approach to rule evaluation that is able to operate under such uncertain conditions and evaluate it by applying it to the case of facial expression analysis. Our approach has a possibilistic, rather than probabilistic, flavor.

Keywords: fuzzy rules, uncertainty, possibility, facial expression analysis.

1 Introduction

Fuzzy rules and fuzzy rule base systems have been used extensively in the past in the design and implementation of expert systems, as they provide for a very intuitive way for expert users to formalize knowledge that can be then utilized to drive an automated intelligent system. The result is systems that can process complex inputs in a very short time and respond accordingly. In cases where response time is critical, systems based on fuzzy rules are often considered as the obvious choice. Measurements received from sensors are mapped to high level linguistic variables, which are then used in order to provide for a quick approximation of the optimal response [2].

The only required assumptions are that the desired response is a continuous function of the considered inputs and of course that the inputs are available. Although these seem to be quite reasonable and relaxed assumptions, cases exist where all of these assumptions cannot be met: when the inputs of an expert system are obtained as the output of another system, as is, for example, an automated
image processing and facial expression analysis system, these inputs might be in some cases unavailable or uncertain [1].

In this paper, driven by such real life problems where the aforementioned situation is observed thus making conventional rule base systems inadequate, we propose a novel fuzzy rule evaluation model that provides for the consideration of uncertain input. Specifically, in section 2 we present our methodology for fuzzy rule evaluation under uncertainty; we explain how this evaluation model produces results that have a possibilistic nature, which also helps tackle the issue of concurrent activation of contradicting rules. In section 3 we apply the proposed approach in the field of facial expression analysis and present the acquired results.

2 Rule evaluation scheme

2.1 Conventional rule evaluation

Fuzzy rule base systems are expert systems that typically contain knowledge in the form of rules such as the following:

\[
\text{IF } x_1 \text{ AND } x_2 \text{ AND } \ldots \text{ AND } x_n \text{ THEN } y
\]  

(1)

where \(y\) is the consequent of the rule and \(x_1, x_2, \ldots, x_n\) are the antecedents of the rule. For example, a rule that could be contained in a fuzzy rule base system is the following:

IF temperature IS high_temp
AND humidity IS high_hum
THEN it_feels_hot IS true

In the temperature example provided above measurement \text{temperature} of the temperature is assumed to be available with absolute precision, so that the membership function can be applied. Although this is a reasonable assumption for sensor driven fuzzy systems, systems that are driven by complex, imprecise and uncertain output cannot be assumed to fulfill it. In many real life problems, such as facial expression estimation, a number of delicate issues has to be considered, such as:

- the case of antecedent values that cannot be estimated,
- the case of antecedents estimated with a low degree of confidence and
- the activation of contradicting rules.

A conventional approach to the evaluation of fuzzy rules of the form described in equation 1 is as follows:

\[
y = t(x_1, x_2 \ldots, x_n)
\]  

(2)

where \(t\) is a fuzzy \(t\)-norm, such as the minimum, the algebraic product, the bounded sum, a Yager norm and so on. Another well known approach in rule evaluation is described in [2] and utilizes a weighted sum instead of a \(t\)-norm in
order to combine information from different rule antecedents. Both approaches are well studied and established in the field of fuzzy automatic control. Still, they are not adequate for the case of problems such as the ones considered herein: their main disadvantage is that they assume that all antecedents are known, i.e. that all features are measured successfully and precisely. In the case of facial expression estimation, for example, feature points may well be estimated with a very low confidence, or not estimated at all, due to low video quality, speech interference, occlusion, noise and so on.

Thus, a more flexible rule evaluation scheme that is able to incorporate this uncertainty is required for this and similar cases. Moreover, the second one of the conventional approaches, due to the summation form, has the disadvantage of possibly providing a highly activated output even in the case that an important antecedent is known to be missing; this is clearly not suitable for problems such as the one examined in this paper; for example, the non activation of a facial animation parameter automatically implies that the expression profiles that require it are not activated either. Therefore, the chosen rule evaluation scheme should be a generalization of the \( t \)-norm based conventional approach of equation 2.

### 2.2 Possibilistic rule evaluation

In the \( t \)-norm operation described in equation 2, antecedents with lower values affect most the resulting value of \( y \), while antecedents with values close to one have trivial and negligible affect on the value of \( y \). Having that in mind, we can demand that only antecedents that are known with a high confidence will be allowed to have low values in that operation. More formally, we demand that the degree \( k(x) \) to which antecedent \( x \) is considered is low when the confidence \( x_{\text{con}} \) with which the value of is known is high and the value of \( x \) is low; depending on the type of application, the degree of confidence may be either provided directly by the sensor or, most probably, estimated via some step of input validation against known criteria. This can be expressed as:

\[
c(k(x)) = t(x_{\text{con}}, c(x))
\]

where \( c \) is a fuzzy complement. Applying de Morgan’s law we have that the degree \( k(x) \) to which antecedent \( x \) is considered is:

\[
k(x) = u(c(x_{\text{con}}), x)
\]

It is easy to see that equation 4 satisfies the desired marginal conditions:

- when \( x \to 1 \), then \( c(x) \to 0 \) and \( k(x) \to x \), i.e. the antecedent is considered normally, while
- when \( x \to 0 \), then \( c(x) \to 1 \) and \( k(x) \to 1 \), i.e. the antecedent is not allowed to affect the overall evaluation of the rule.
The formula assumed in this discussion for the overall evaluation is the one followed by the conventional approach, with the exception that antecedents participate with their considered values:

\[ y = t(k(x_1), k(x_2) \ldots k(x_n)) \] (5)

It is easy to see that in the case that all antecedents are known with a confidence of 1 the rule will be evaluated in the same way as in the conventional methodology. When, on the other hand, all antecedents are known with a confidence of zero, i.e. when no information is available, the rule will be evaluated with a degree of 1. Thus, the activation level of a rule with this approach can be interpreted in a possibilistic manner, i.e. it can be interpreted as the degree to which the corresponding output is possible, according to the available information; in the literature, this possibilistic degree is referred to as plausibility [2].

As far as the confidence in the calculated output is concerned, in the conventional approach we either always assume total confidence in the output, which originates from the assumption that all inputs are precisely known. In the extended approach followed herein, where we accept that one or more of the required rule antecedents may be unknown or known with a confidence other than zero, it does not make sense to always assume total confidence in the computed activation level. Quite the contrary, the calculated activation level is only complete in information when associated with a corresponding degree of confidence. The confidence is determined by the confidence values of the utilized inputs, i.e. by the confidence values of the rule antecedents, as follows:

\[ y_{\text{con}} = x_{1-\text{con}} + x_{2-\text{con}} + \ldots + x_{n-\text{con}} \] (6)

The definition of \( y_{\text{con}} \) in this manner has the desired effect that is equivalent to the complete lack of information; this property is essential in possibilistic reasoning [2].

### 2.3 On the possibilistic flavor of the evaluation

When using the conventional rule evaluation methodology of 2, it is not rare to obtain crisp results by selecting the rule that was activated to the highest degree and ignoring the rest. This means that the conventional interpretation of the evaluation of the fuzzy rule has a probabilistic nature. This can be seen more clearly from the fact that:

- the rule with the highest activation is considered most probable than the others and
- cases where two rules are activated to almost equal degrees are not considered “clear” as both rules are considered almost equally probable.

On the other hand, it is worth noting that the output of a fuzzy rule base system, although has a probabilistic flavor, cannot be used as a formal probability
measure as it does not meet the axiomatic probability definition. For example, the sum of the activation of contradicting rules is not guaranteed to be less than unity. Similarly, we cannot claim that the output of the proposed possibilistic rule evaluation meets all the formal requirements to be named possibilistic. Still, it is clear that it has a possibilistic nature.

Thus, the activation level of the rule has a possibilistic interpretation and corresponds to the plausibility of the rule. In order, of course, to have a complete possibilistic representation of the rule evaluation, together with the plausibility of the rule we need to estimate the corresponding belief, i.e. the degree to which available evidence suggests that the output case described by the rule is verified by the available input.

The belief measure should be high when sufficient information is available during the evaluation of the rule, and that information verifies the output case described by the rule. The amount of information that is available during the evaluation of the rule is provided by the calculated confidence value, while the degree to which this information verifies the output case described by the rule is provided by the activation level. Thus, the complete possibilistic representation of the calculated output is provided as:

$$Bel = t(y, y_{con})$$  \hspace{1cm} (7)

$$Pl = y_{con}$$  \hspace{1cm} (8)

The extreme cases are

- $Bel = Pl = 1$, which occurs when $y = y_{con} = 1$ and implies absolute confidence that the specific emotional state is the one perfectly matching the observed face
- $Bel = Pl = 0$, which occurs when $y = 0$ and implies absolute confidence that the specific profile is not one matching the observed face and
- $Bel = 0$, $Pl = 1$, which occurs when $y_{con} = 0$ and implies absolute ignorance

The case of activation of multiple and contradicting rules of the rule base is not an issue for this approach. In that case, it is expected that confidence values will be low, which can be interpreted as the case in which, due to poor input, more than one possible output cases cannot be ruled out. Still, the belief that they should indeed be activated, as reported by equation 7, will be low.

3 Case study in facial expression analysis

The methodology presented in this paper has been applied in the framework of ERMIS EU funded project where facial images are analyzed automatically in order to estimate firstly the expression and then the emotion of the observed user [5].

The images that ERMIS considers are real life, low quality, low resolution, variable lighting and frequent occlusion image sequences where conventional facial analysis methodologies typically fail. Interesting facial features, such as the
eyes and the mouth are located on the face and their boundaries are estimated. Then, based on the positions of feature boundaries, a set of relevant rules estimating the user’s emotion are evaluated. In order to overcome the difficulties imposed by the low quality of the input frames ERMIS utilizes:

- multiple methodologies are combined towards the estimation of the facial features and
- the certainty with which each feature has been detected is estimated using statistical anthropometric data [4] and considered in the consequent step of emotion estimation based on the methodology presented herein.

In order to experimentally validate the efficiency of our methodology we have implemented two versions of the emotion estimation module, one utilizing the conventional evaluation model of equation 2 and one using the proposed model of equations 7 and 8.

The proposed methodology has achieved 78.4%, in comparison to 65.1% of the conventional methodology that does not consider degrees of confidence, by successfully dealing with many cases in which some facial features have been imperfectly detected. An indicative example is presented in figure 1. Here, due to a mistake in the detection of the left eye, the estimation of many facial parameters has been unsuccessful. As a result, none of the rules in the knowledge base is activated when using the conventional rule evaluation approach that does not consider degrees of confidence. On the contrary, the following rule is activated when using the proposed methodology.

IF close_left_eye IS close_left_eye_low
AND close_right_eye IS close_right_eye_low
AND raise_left_inner_eyebrow IS raise_left_inner_eyebrow_high
AND raise_right_inner_eyebrow IS raise_right_inner_eyebrow_high
AND raise_left_medium_eyebrow IS raise_left_medium_eyebrow_high
AND raise_right_medium_eyebrow IS raise_right_medium_eyebrow_high
AND raise_left_outer_eyebrow IS raise_left_outer_eyebrow_high
AND raise_right_outer_eyebrow IS raise_right_outer_eyebrow_high
THEN output IS quadrant_1

The activation acquired for the above rule following the proposed approach is $y = 0.3015$ and the confidence is $y_{con} = 0.716673$. Using formulas 7 and 8 this is interpreted as $Bel = 0.216077$ and $Pl = 0.216077$ by assuming product as the $t$-norm of choice. The meaning of these values is that there is some limited evidence that this rule should be activated ($Bel$) and that additionally that this is not contradicted by the available information ($Pl$) which is exactly the case given the uncertainty with which the facial feature positions are known.

4 Conclusions

In this paper we have proposed a novel scheme for the evaluation of fuzzy rules in a framework of existing and measurable uncertainty. The proposed approach utilizes degrees of confidence in order to determine the extend to which each bit of information should be allowed to affect the overall outcome of the evaluation process. The result is a possibilistic evaluation of fuzzy rules; this kind of evaluation, as indicated by the experimental results, is more suitable for operation under uncertainty than the conventional approach.

The approach has been applied in the framework of the ERMIS system. Through this application we have experimentally verified that the possibilistic nature of the evaluation is indeed capable of successfully handling uncertainty in its input and of incorporating this uncertainty in its final output in the form of joined belief and plausibility degrees.

This being a first approach to this topic, we do not feel that the issue is closed with the current work. As future extensions, beyond the application of the proposed methodology to different practical problems, we also consider the incorporation of the rule evaluation approach in a neurofuzzy structure and the specification of the training of such a structure based on uncertain data.

References